Lesson 5: Points of Concurrencies

Student Outcome

- Students become familiar with vocabulary regarding two points of concurrencies and understand why the points are concurrent.

Lesson Notes

Lesson 5 is an application lesson of the constructions covered so far.

In the *Opening Exercise*, students construct three perpendicular bisectors of a triangle but this time using a makeshift compass (a string and pencil). Encourage students to take note of the differences between the tools and how the tools would alter the way the steps would be written.

The *Discussion section* address vocabulary associated with points of concurrencies. The core of the notes presents *why* the three perpendicular bisectors are in fact concurrent. Students are then expected to establish a similar argument that explains why the three angle bisectors of a triangle are also concurrent.

The topic of points of concurrencies is a nice opportunity to consider incorporating geometry software if available.

Classwork

Opening Exercise (7 minutes)

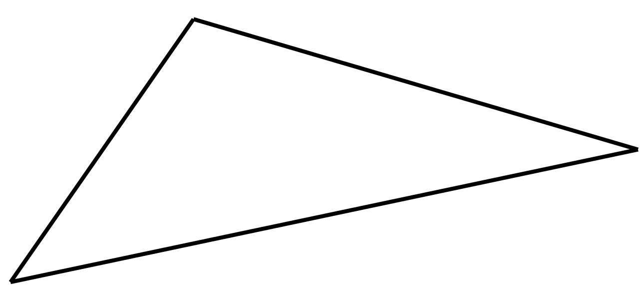
Students use an alternate method of construction on Lesson 4, Problem Set #1.

MP.5

Opening Exercise

You will need **a make-shift compass made from string and a pencil.**

Use these materials to construct the perpendicular bisectors of the three sides of the triangle below (like you did with Problem Set # 2).



How did using this tool differ from using a compass and straightedge? Compare your construction with that of your partner. Did you obtain the same results?

Discussion (38 minutes)

Discussion

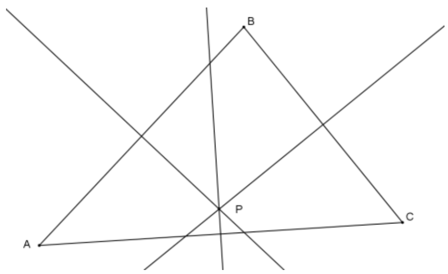
When three or more lines intersect in a single point, they are *concurrent*, and the point of intersection is the *point of concurrency*.

You saw an example of a point of concurrency in yesterday's problem set (and in the Opening Exercise above) when all three perpendicular bisectors passed through a common point.

The point of concurrency of the three perpendicular bisectors is the *circumcenter of the triangle*.

The circumcenter of $\triangle ABC$ is shown below as point P.

Have students mark the right angles and congruent segments (defined by midpoints) on the triangle.



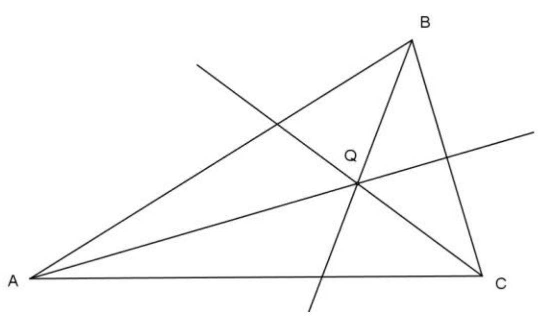
The question that arises here is WHY are the three perpendicular bisectors concurrent? Will these bisectors be concurrent in all triangles? To answer these questions, we must recall that all points on the perpendicular bisector are equidistant from the endpoints of the segment. This allows the following reasoning:

1. **P is equidistant from A and B since it lies on the *perpendicular bisector* of $\overline{AB}$.**
2. **P is also *equidistant* from B and C since it lies on the perpendicular bisector of $\overline{BC}$.**
3. **Therefore, P must also be equidistant from A and C.**

Hence, $AP = BP = CP$, which suggests that P is the point of *concurrency* of all three perpendicular bisectors.

You have also worked with angles bisectors. The construction of the three angle bisectors of a triangle also results in a point of concurrency, which we call the *__incenter__*.

Use the triangle below to construct the angle bisectors of each angle in the triangle to locate the triangle's incenter.



Have students label the congruent angles formed by the angle bisectors.

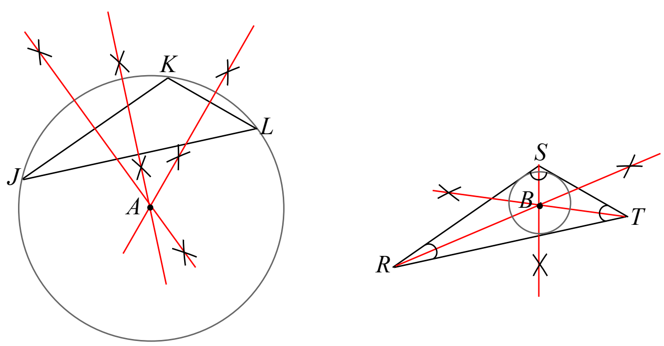
> *Note to Teacher:*
>
> For the first question, students do not need to re-explain constructions they have established prior to the current construction.

1. **State precisely the steps in your construction above.**
   1. ***Construct the angle bisectors of $\angle A$, $\angle B$, and $\angle C$.***
   2. ***Label the point of intersection Q.***

2. **Earlier in this lesson, we explained why the perpendicular bisectors of the sides of a triangle are always concurrent. Using similar reasoning, explain clearly why the angle bisectors are always concurrent at the incenter of a triangle.**

   Any point on the angle bisector is equidistant from the rays forming the angle. Therefore, since point Q is on the angle bisector of $\angle ABC$, it is equidistant from $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Similarly, since point Q is on the angle bisector of $\angle BCA$, it is equidistant from $\overrightarrow{CB}$ and $\overrightarrow{CA}$. Therefore, Q must also be equidistant from $\overrightarrow{AB}$ and $\overrightarrow{AC}$, since it lies on the angle bisector of $\angle BAC$. So Q is a point of concurrency of all three angle bisectors.

3. **Observe the constructions below. Point A is the *circumcenter* of $\triangle JKL$. (Notice that it can fall outside of the triangle.) Point B is the *incenter* of $\triangle RST$. The circumcenter of a triangle is the center of the circle that circumscribes that triangle. The incenter of the triangle is the center of the circle that is inscribed in that triangle.**



   On a separate piece of paper, draw two triangles of your own below and demonstrate how the circumcenter and incenter have these special relationships.

   Answers will vary.

4. **How can you use what you have learned in Problem 3 to find the center of a circle if the center is not shown?**

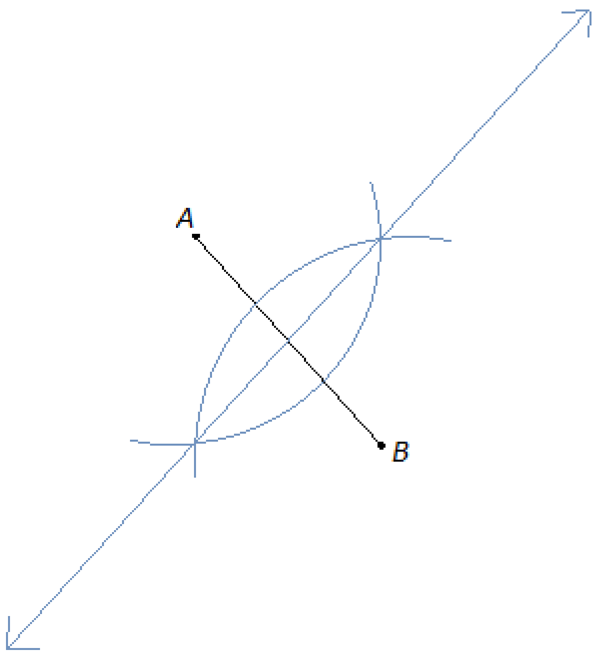
   Inscribe a triangle into the circle and construct the perpendicular bisectors of at least two sides. Where the bisectors intersect is the center of the circle.

Closing

Inform students about the topic shift to *unknown angle problems and proofs* for the next six lessons. The Lesson 5 *Problem Set* is a preview for Lessons 6–11, but is based on previously taught geometry facts.

Problem Set Sample Solutions

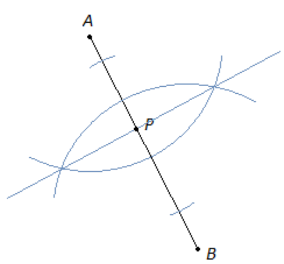
1. Given line segment AB, using a compass and straightedge construct the set of points that are equidistant from A and B.

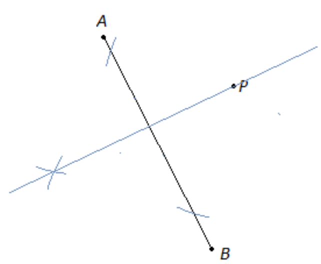


What figure did you end up constructing? Explain.

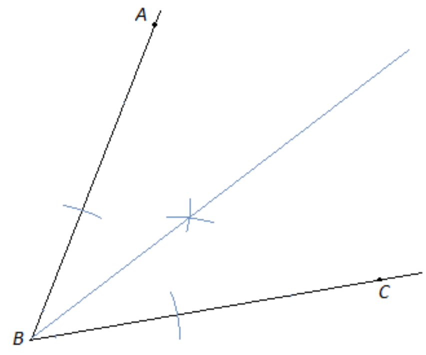
I ended up drawing the perpendicular bisector of the segment AB. Every point on this line is equidistant from the points A and B.

2. For each of the following, construct a line perpendicular to segment AB that goes through point P.





3. Using a compass and straightedge, construct the angle bisector of $\angle ABC$ shown below. What is true about every point that lies on the ray you created?



Every point on the ray is equidistant from ray BA and ray BC.